Algebras and Languages for Molecular Programming

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Torino, 2010-03-24 http://lucacardelli.name

Smaller and Smaller

Dec. 23, 1947. John Bardeen and Walter Brattain show the first working transistor.

Sep. 1958. Jack Kilby builds the first integrated circuit.

50 years later Jan. 2010. Intel and Micron announce 25nm NAND flash.

Dec. 24, 2009. Working transistor made of a single molecule.

Observation of molecular orbital gating. *Nature*, 2009; 462 (7276): 1039

<10 iterations of Moore's Law left! The race is on for *molecular scale integrated circuits*.



Molecular Transistor

Placement and orientation of individual DNA shapes on lithographically patterned surfaces. Nature Nanotechnology 4, 557 – 561 (2009).

Building The Smallest Things

- How do we build structures that are by definition smaller than your tools?
- Basic answer: you can't. Structures (and tools) should build themselves!
- By programmed self-assembly.





Molecular IKEA

- Nature can self-assemble. Can we?
- *"Dear IKEA, please send me a chest of drawers that assembles itself."*
- We need a magical material where the pieces are pre-programmed to fit into to each other.
- At the molecular scale many such materials exist; let's pick one...



DNA



GC Base Pair Guanine-Cytosine





TA Base Pair Thymine-Adenine

Interactive DNA Tutorial

(http://www.biosciences.bham.ac.uk/labs/minchin/tutorials/dna.html)



Sequence of Base Pairs (GACT alphabet)

Robust, and Long

• DNA in each human cell:

- 3 billion base pairs
- o 2 meters long, 2nm thick
- \circ folded into a 6µm ball
- o 750 MegaBytes
- A huge amount for a cell
 - Every time a cell replicates it has to copy *2 meters of DNA* reliably.
 - To get a feeling for the scale disparity, compute:
- DNA in human body
 - o 10 trillion cells
 - 133 Astronomical Units long
 - o 7.5 OctaBytes
- DNA in human population
 20 million light years long



DNA wrapping into chromosomes



Andromeda Galaxy 2.5 million light years

Zipping Along

• DNA can support structural and computational complexity.



DNA replication in *real time*

In Humans: 50 nucleotides/second Whole genome in a few hours (with parallel processing)

> In Bacteria: 1000 nucleotides/second (higher error rate)



DNA transcription in *real time*

RNA polymerase II: 15-30 base/second

Drew Berry http://www.wehi.edu.au/wehi-tv

Nanoscale Engineering

Sensing

Reacting to forcesBinding to molecules

Actuating

- Releasing moleculesProducing forces
- Constructing
 - o Chassis
 - o Growth

Computing

- Signal Processing
- Decision Making



Nucleic Acids can do all this. And interface to biology.

Hybridization



- Strands with opposite orientation and complementary base pairs stick to each other (Watson-Crick duality).
- This is all we are going to use
 - We are not going to exploit DNA replication, transcription, translation, restriction and ligation enzymes, etc., which enable other classes of tricks.



Hybridization Tricks



Constructing

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In nature, crosslinking is deadly (blocks DNA replication).



In engineering, crosslinking is the key to using DNA as a construction material.

DNA Tiling







Pankhudi

2D DNA Lattices



Chengde Mao Purdue University, USA



N-point Stars



3D DNA Structures



Ned Seeman NYU





3D Cyrstal



AndrewTuberfield Oxford





I : Base pair

Tetrahedron



S.M. Douglas, H. Dietz, T. Liedl, B. Högberg, F. Graf and W. M. Shih Self-assembly of DNA into nanoscale three-dimensional shapes, Nature (2009)

DNA Origami

- Folding long (7000bp) naturally occurring (viral) ssDNA
- By lots of short 'staple' strands that constrain it



PWK Rothemund, Nature 440, 297 (2006)

Black: long viral strand Color: short staple strands



DNA Origami





Paul Rothemund's "Disc with three holes" (2006)



This means we can already selfassemble meso-scale structures.

Paul W K Rothemund California Institute of Technology

DNA Circuit Boards

DNA-wrapped nanotubes 6 nm grid of individually addressable pixels

PWK Rothemund, Nature 440, 297 (2006)



European Nanoelectronics Initiative Advisory Council

"What we are really making are tiny DNA circuit boards that will be used to assemble other components." *Greg Wallraff, IBM*



Sensing

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Aptamers

• Artificially eveloved DNA molecules that stick to anything you like (highly selectively).



Pathogen Spotlights

- DNA aptamer binds to:
 - A) a pathogen
 - B) a molecule our immune system already hates and immediately removes (eats) along with anything attached to it

- Result: instant immunity
 - Mice poisoned with Anthrax plus aptamer (100% survival)
 - Mice poinsoned with Anthrax (not so good)





Time (days)

Kary Mullis (incidentally, also Nobel prize for inventing the Polymerase Chain Reaction)



Computing

Basic Steps

Compositionality

- Sensors and Actuators at the 'edge' of the system
 They can use disparate kinds of inputs (sensors) and outputs (actuators)
- The 'kernel' of the system computes
 - <u>Must</u> use uniform inputs and outputs
- Compositionality in the kernel
 - Supporting 'arbitrary' computing complexity
 - The output of each computing components must be the same kind of 'signal' as the input
 - If the inputs are voltages, the outputs must be voltages
 - If the inputs are DNA, the outputs must be DNA
- Central design question
 - What should our signals (not components!) be?
 - Then design components that manipulate those signals.

Rules of the Game

Short complementary segments hybridize reversibly



Long complementary segments hybridize irreversibly



DNA Strand Displacement

- Short strand (toehold): reversible binding
- Long strand (body): irreversible binding



Failed Strand Displacement

• What if the input does not match the gate?





Failed Strand Displacement d⊥

Failed Strand Displacement



Failed Strand Displacement d⊥


Failed Strand Displacement

Hence an incorrect binding will undo

• That's why toeholds must bind reversibly



- Matching depends on the long segment only
 - Strand displacement succeeds iff the whole long segment matches
 - The address space is determined by the size of the long segment, which is unbounded (not by the size of the toehold)
 - The toehold is just a 'cache' of the address



Computing

Implementing "Arbitrary" Computing Functions

What does DNA Compute?

• Electronics has *electrons*

- All electrons are the same: you can only count them
- *Few* electrons = False; *lots* of electrons = True
- But Boolean Logic is only a necessary evil to build symbolic computation

DNA computing has symbols (DNA words)

- DNA words are not all the same
- Symbolic computation on abstract signals can be done *directly*
- Signals are presented concurrently (in a soup)
- No requirement to do Boolean Logic
- Then, what are our 'gates' (if not Boolean?)
 - Theory of Concurrency
 - Process Algebra as the "Boolean Algebra" of DNA Computing

Signals

- A signal is the representation of an abstract event
 - E.g. generated by a sensor
 - E.g. accepted by an effector
 - We are not limited to true/false
- 3-domain signals
 - x_h: hystory (ignore)
 - \circ x_t: toehold (binding)
 - x_b: body (recognition)



• Signals (single stranded DNA) are prepared by (artificial) DNA synthesis



 Gates are prepared by self-assembly from singlestranded DNA that is synthesized











































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General n-Join/m-Fork Gate


Strand Algebra $x_1 \mid .. \mid x_n \mid [x_1,..,x_n].[y_1,..,y_m] \rightarrow y_1 \mid .. \mid y_m$

• Join + Fork + Populations = (Stochastic) Petri Nets

 \square



Gate Design Verification

Active garbage

- The active join residuals slow down the performance of following joins.
- $\circ \rightarrow$ Add a garbage collector to remove the active residuals.

Interference between gates

- The join garbage collector interferes with the fork gate.
- $\circ \rightarrow$ Modify the fork gate to remove the interference.

• What else could go wrong?

- Endless possibilities.
- → Prove that the fork/join gate structures correctly implement fork/join in all larger circuits.



Actuating

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Polymerization Motor



An autonomous polymerization motor powered by DNA hybridization

SUVIR VENKATARAMAN¹, ROBERT M. DIRKS¹, PAUL W. K. ROTHEMUND^{2,3}, ERIK WINFREE^{2,3} AND NILES A. PIERCE^{1,4,*}

Rickettsia (spotted fever)





Directional Actin Polymerization Associated with Spotted Fever Group Rickettsia Infection of Vero Cells ROBERT A. HEINZEN, STANLEY F. HAYES, MARIUS G. PEACOCK, AND TED HACKSTADT*



Curing

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A Doctor in Each Cell



William Silverman



Tools

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Sequence Design



Visual DSD A Strand Displacement Simulator

Matthew Lakin, Simon Youssef, Andrew Phillips http://lepton.research.microsoft.com/webdna/

Syntax



A programming language for composable DNA circuits

Andrew Phillips^{*} and Luca Cardelli



Dynamics



Initial Species

al DSD - loca	lhost						NAME AND	Castle Village	All Providences							
es:	•	Solve Si	imulate	Pause	Rules: Defa	ult 🔻	Simulation:	Stochastic	 View 	v options: 💌	Unproductiv	/e: 📃 I	Leaks: 📃	Domains: 📃	v0.12-20100224-152	1 Upd
de D	A Initial)		Species	Reactions	Graph	Text	SBML	Domains	Table	Pl	ot			
					Zoom	1	09 % Fit	Save								
2 0																
2	3 4 5	2	3	4												
		4	5													
					1											
					4											

Reaction Graph



Simulation



Abstract Reactions



Detailed Reactions



Detailed Leak Reactions!



Just-in-Time Simulation



DNA Sequences

🖳 Visual DSD - localhost	- 0 x
Examples: Solve Simulate Pause Rules: Default Simulation: Stochastic View options: Unproductive: Leaks: Domains: View options: Unproductive: Leaks: Domains: View options:	0.12-20100302-1033 Update
Code DNA Taitial Session Pagetions Craph Taut SEMI Domains Takin Plat	
Code Dive Initian Species Reactions Graph Text Specie Domains Table Plot	
Check sequences Zoom Zoom	
TOEHOLD SEQUENCES 3^> TATTCC	
GCTA 5^> GCTA	
1> CCCTTTACATTACATAACAA	
CCTACG 4> CCCTTTTCTAAACTAAACAA	
GACA	
ACCT	
CACACA	
AGAC	
SPECIFICITY SEQUENCES	
СССААААСААААСАА	
СССТТТТСТАААСТАААСАА	
CCCTTTACATTACATACAA	
СССТТААСТТААСАААТСТА	
CCCTATTCAAATCAA	
СССТААТСТААТСАТААСТА	
CCCTAAACTTATCTAAACAT	
CCCATTACTAATCAATTCAA	
CCCATATCTATACATTACAA	
CCCATACTATICTAACATACTA	
CCCAAATCTTAACTATACTA	
CCATATCCATAACTTTACAA	
CCATTACCATATCTTATCAT	

Final DNA Circuit



Next-Day Oligos!



Place Order NOW!



DNA by Mail



lt runs!

• (Nothing to do with us.)

Engineering Entropy-Driven Reactions and Networks Catalyzed by DNA David Yu Zhang, et al. Science **318**, 1121 (2007); DOI: 10.1126/science.1148532



DNA Compilation

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Compilers



Intermediate Languages Boolean Petri Networks Nets **Front End** Intermediate Strand The algebra of fork Language Algebra and join gates **Back End**





Compiling Abstract Machines

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Boolean Networks

Boolean Networks to Strand Algebra

 $([a_F, b_F].c_T)^* |$ ([a_F,b_T].c_T)* | ([a_T,b_F].c_T)* | ([a_T,b_T].c_F)* | a_F | b_T

This encoding is *compositional*, and can encode *any* Boolean network:

- multi-stage networks can be assembled (combinatorial logic)
- network loops are allowed (sequential logic)

Petri Nets

Petri Nets to Strand Algebra

Transitions as Gates Place markings as Signals

([p₁,p₂].[p₃,p₄])*| p₁|p₁|p₄

Chemical Reaction Networks

Implementing an arbitrary finite chemical system in DNA with asymptotically correct kinetics Soloveichick & al. DNA 15

Species become signals Reactions become gates

$A + B \rightarrow C + D \qquad \Rightarrow \qquad [A,B].[C,D]$

Interacting Automata

([A,B].[B,B])* | ([B,C].[C,C])* | ([C,A].[A,A])* | A | A | B | C

This is a uniform population of identical automata, but heterogeneous populations of interacting automata can be similarly handled.

Interacting Automata

([A,B].[B,B])* | ([B,C].[C,C])* | ([C,A].[A,A])* | A | B | B | C

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Interacting Automata





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This is a uniform population of identical automata, but heterogeneous populations of interacting automata can be similarly handled.



- compile(0) = empty solution
- compile(P | P') = mix(compile(P), compile(P'))
- compile(P*) = population(compile(P))

And finally...

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Summary

- Abstract Machines to Strand Algebra
 - Or other intermediate language
- Strand Algebra to DSD
 - Or other structural language

• Simulation, analysis, etc.

o Iterate a lot

DSD to Sequences

- E.g. NuPack, or pre-build strand libraries
- Sequences to DNA
 - Web order
- DNA experiments
 - Fairly basic wet lab
- Deployable Nanotech

Conclusions

Nucleic Acids

• Programmable matter

DNA Strand Displacement

• A computational mechanism at the molecular level

DNA as a Compilation Target for Abstract Machines

- Abstract Machines (Boolean Networks, Petri Nets, Interacting Automata)
- Intermediate languages (Strand Algebra, Strand Displacement Language).
- DNA sequence generation.

Tools

- Thermodynamic analysis.
- Reaction graph generation.
- Simulation.
- Verification (not yet).

Acknowledgments



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 - o Erik Winfree, Caltech
 - o Bernard Yurke, Boise State
 - o Wikipedia
 - YouTube

David Soloveichik

